# REPORT ON THE RECENT SERIES OF PREPRINTS BY K. JOSHI

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## Section 1: General Overview

Over the past few years, K. Joshi has released a series of preprints, culminating in the following two recently released preprints:

[CnstIII] K. Joshi, *Construction of Arithmetic Teichmuller Spaces III: A 'Rosetta Stone' and a proof of Mochizuki's Corollary 3.12* (January 25, 2024 version), preprint.

[CnstIV] K. Joshi, *Construction of Arithmetic Teichmuller Spaces IV: Proof of the abc-conjecture* (March 10, 2024 version), preprint.

Since this series of preprints contains numerous assertions concerning *inter-universal Teichmüller theory*, a theory exposed in the surveys [Alien], [EssLgc] [cf. also the original papers [IUTchI-IV], [ExpEst]], I have been requested by various people to make public my understanding concerning the content of this series of preprints. In a word, my *short answer* to such requests may be summarized as follows:

(ShtAns) although Joshi, in this series of preprints, makes references to and often uses certain portions of the terminology and notation of inter-universal Teichmüller theory, it is *conspicuously obvious* to any reader of these preprints who is equipped with a solid, rigorous understanding of the actual mathematical content of inter-universal Teichmüller theory that the author of this series of preprints is **profoundly ignorant** *of the actual mathematical content of inter-universal Teichmüller theory*, and, in particular, that this series of preprints does **not contain**, at least from the point of view of the mathematics surrounding inter-universal Teichmüller theory, **any meaningful mathematical content whatsoever**.

To my knowledge,

(NotPb) *none of these preprints has been published in an internationally recognized mathematical journal.*

Nevertheless, in accordance with the *spirit* of [i.e., as opposed to the strictly interpreted ethical duties implied by] the passage [namely, Article (6.)] of the subsection entitled "Responsibilities of authors" of the Code of Practice of the European Mathematical Society [cf. [EMSCOP]] that I often quote — i.e.,

> *Mathematicians should not make public claims of potential new theorems or the resolution of particular mathematical problems unless they are able to provide full details in a timely manner.*

— it should be recalled that

(WrEx) it is of *fundamental importance* to the sound development of the field of mathematics, especially from a *historical point of view*, to produce **detailed, explicit, mathematically substantive**, and **readily accessible written expositions** of the **logical structure** underlying publicly articulated mathematical assertions,

i.e., as opposed to resorting to justifications of such mathematical assertions via invocations of such *mathematically meaningless* — and indeed *largely psychologically/socially/politically based* — notions as the notion of *"common sense"* [cf. the discussion of [EssLgc], §1.3, §1.5, §1.8]. The present report may be understood as a written exposition in the spirit of (WrEx).

In this context, it should be recalled [cf., e.g., [Rpt2023]] that the original papers on inter-universal Teichmüller theory, i.e., [IUTchI-IV], as well as a sequel [ExpEst] to these original papers, were published a number of years ago in leading international journals. Moreover, there is by now a quite substantial body of mathematicians who are not only thoroughly familiar with the original theory [i.e., of [IUTchI-IV]], but have also been quite actively engaged for a number of years now in research on further enhanced versions of inter-universal Teichmüller theory. Over the past few years, and especially over the past few months, I consulted with many of these researchers concerning the issue of what would be the optimal way to respond to Joshi's series of preprints. The present report may be understood, to a substantial extent, as the product of these consultations. The reactions that I met with during these consultations may be summarized as follows:

(Reac1) First of all, there was an **entirely unanimous consensus** that Joshi's series of preprints was **obviously mathematically meaningless**, and that it was obvious that he did not have any idea what he was talking about.

(Reac2) In light of the blatant obviousness underlying (Reac1), many consultees were *not interested in investing the time and effort necessary to discuss Joshi's series of preprints in detail* and indeed *strongly encouraged me to* **simply ignore** *them* as well.

(Reac3) On the other hand, in response to (Reac2), I emphasized [cf. the above discussion surrounding (WrEx)] the importance of producing *detailed, explicit, mathematically substantive, and readily accessible written expositions* of the **mathematical content** underlying the unanimous reaction of researchers of the inter-universal Teichmüller theory community [cf. (Reac1)].

(Reac4) In the context of (Reac3), I would also emphasize another important aspect of (Reac3), namely, that the **"exercise"** of making the mathematical content mentioned in (Reac3) *explicit*, in accordance with (WrEx), should be understood as a **valuable pedagogical tool**, both for seasoned researchers and novices in inter-universal Teichmüller theory, for deepening one's understanding of the mathematics involved [cf. the discussion of [EssLgc], §1.6, §1.7, §1.8, §1.10, §3.1].

Finally, in the context of the theme of *"computers and inter-universal Teichmüller theory"* discussed in [EssLgc], §1.12, I should mention that I could not help but notice the following *stimulating interconnections* between this theme of "computers and inter-universal Teichmüller theory" and Joshi's series of preprints:

(AI1) When browsing through Joshi's series of preprints, i.e., whose content consists of a sort of *rough concatenation of various "fragments"* of inter-universal Teichmüller theory that is nonetheless *devoid of any substantive mathematical understanding* [cf. (ShtAns)], I could not help but be reminded of the so-called **"hallucinations"** produced by *artificial intelligence* algorithms, such as ChatGPT, i.e., which are synthesized precisely by means of various *mechanically searched contextual concatenations* that are entirely *devoid of any genuine "human" understanding of the actual content of the text involved.*

(AI2) The observation of (AI1) suggests that in the future, it is quite possible that the production, or indeed *mass production* (!), of similar **"pseudo-mathematical texts"** by articial intelligence algorithms may become more widespread and, in particular, *pose a substantial threat* to the sound development of the field of mathematics by sewing the seeds of *entirely unnecessary confusion* in the worldwide mathematical community concerning established mathematical theories.

(AI3) On the other hand, it is also quite possible that in the future, the *"exercise"* of producing suitable written expositions in response [cf. (Reac4)] — which can often require a *quite substantial investment of time and effort* of the sort that many researchers in the field, who are quite busy with their own research projects, are simply not willing to make [cf. (Reac2)] — could be performed, as least partially, via *artificial intelligence algorithms* for the **automated parsing of texts**, so that researchers would not need to invest the time and effort necessary to do this themselves. Indeed, one important aspect of the **valuable pedagogical** nature of the *"exercise"* of producing suitable written expositions in response to manuscripts such as [CnstIII], [CnstIV] [cf. (Reac4)] lies precisely in the fact this sort of "exercise" may be thought of as a sort of *initial, preparatory step*, relative to the *ultimate goal of designing such automated text parsing algorithms.*

(AI4) In the context of the issue of substantial investments of time and effort [cf. (Reac2), (AI3)], I should also mention that I was *saddened*, as I browsed through Joshi's series of preprints, to contemplate the quite considerable investment of time and effort that Joshi must have put, presumably without any significant use of artificial intelligence algorithms, into writing this series of preprints.

In the context of (AI4), it is also interesting to note that during the past few years during which Joshi wrote this series of preprints,

(TmEff) quite a number of mathematicians were able to study and achieve a *genuine mathematical understanding of inter-universal Teichmüller theory* in the usual, conventional way [often with the help of [Alien], [EssLgc]], while expending surely no more than a *tiny fraction of the time and effort that Joshi must have put into writing this series of preprints during the same time period.*

**Section 2: Local tilts and global arithmetic inequalities**

Joshi's central assertion concerning the mathematical validity of inter-universal Teichmüller theory [cf. [CnstIII], §1] consists of reaffirming the assertions of the **RCS**, or **"redundant copies school"**, concerning the essential logical structure of inter-universal Teichmüller theory. In particular, Joshi denies the mathematical validity of inter-universal Teichmüller theory on the basis of the assertions of the RCS. The confusion surrounding these assertions of the RCS is explained in detail throughout [EssLgc], especially in [EssLgc], §3. Joshi does not, in [CnstIII], [CnstIV], add any new essentially new content to the assertions of the RCS. As discussed throughout [EssLgc], these assertions of the RCS amount to a very **elementary** and **meaningless misunderstanding** that corresponds, in a very precise fashion, to any one of the following well-known elementary examples:

(RC-Ex1) the *"contradiction $T = T^{-1}$"* concerning the standard coordinate on the projective line that arises if one *arbitrarily identifies* the two copies of the affine line that appear in the usual gluing construction of the *projective line*, or, alternatively, the northern and southern hemispheres on the *Riemann sphere* [cf. the discussion of [EssLgc], Example 2.4.7, for more details];

(RC-Ex2) the *"contradiction"* to the effect that the multiplicative map given by raising to the $N$-th power, for $N$ an integer $\geq 2$, on an integral domain of characteristic zero, is a *ring homomorphism* if one *arbitrarily identifies* the domain and codomain of this multiplicative map [cf. the discussion of [EssLgc], Example 2.4.8, for more details];

(RC-Ex3) the *"contradiction"* — i.e., to the effect that a *non-holomorphic* Teichmüller map is *holomorphic* — that arises from *arbitrarily identifying* the two copies of the *complex plane* [regarded, say, as a Riemann surface] obtained by considering the *distinct holomorphic structures* on a *single copy* of the Euclidean plane that arise from *nontrivial Teichmüller deformations* — i.e., of the form

$$\Lambda : \mathbb{C} \ni x + iy \mapsto \lambda \cdot x + iy \in \mathbb{C},$$

for some real number $\lambda > 1$ — as in [EssLgc], Example 3.3.1 [cf. also [EssLgc], Example 3.5.2, (iii)].

In this context, it is perhaps *somewhat ironic* to observe that the *very definition* of the notion of a **tilt**, which plays a fundamental role in [CnstIII], [CnstIV], becomes **invalid/self-contradictory** if one applies the **RCS approach** of *arbitrarily identifying isomorphic objects* in some nontrivial system to the *distinctly labeled copies* of "$\mathcal{O}_k$" that appear in the inverse limit

$$\mathcal{O}_{k^\flat} \quad = \quad \varprojlim \ \mathcal{O}_k$$

used to define the *tilt*. Indeed, this approach which would imply that the above inverse limit is in fact given by the set

$$\{x \in \mathcal{O}_k \mid x^p = x\}$$

— i.e., a set that is *manifestly completely different* from the set "$\mathcal{O}_{k^\flat}$" obtained by taking the inverse limit as in the definition of the tilt [in the conventional way!] with the *distinct labels intact* [cf. the discussion of [EssLgc], Example 3.5.3, (v), for more details, e.g., concerning the notation]. When viewed relative to the analogy constituted by the present discussion of applying the *RCS approach* to the definition of a tilt, Joshi's series of preprints may be understood as

(EpEx) a sort of *epic exercise in mental gymnastics* devoted to showing that a theory based on the **"RCS approach-motivated definition"** of a tilt by some sort of inverse limit of the form

$$\varprojlim \ \mathcal{O}_{k_n}$$

— where the $\{k_n\}_{n \in \mathbb{N}}$ is a collection of $p$-adic fields indexed by the nonnegative integers such that, for distinct nonnegative integers $n$, $m$, the topological fields $k_n$ and $k_m$ are *non-isomorphic* — in fact *satisfies essentially the* **same properties** as the properties asserted in the theory of tilts relative to the conventional definition of "$\mathcal{O}_{k^\flat}$".

Of course, it will be immediately obvious to any mathematician who has a *genuine mathematical understanding* of the conventional theory of tilts that such an epic exercise is nothing more than a **completely meaningless exercise in futility**, and that it would be much more *meaningful* and *productive* [cf. (TmEff)] for the author of such a series of preprints to devote his time and energy to reexamining *why* he arrived at the *[patently erroneous!] conclusion* that the *RCS approach* — i.e., to the effect that **arbitrarily identifying isomorphic objects** in some nontrivial system has **no substantive mathematical/logical effect** on the mathematics involved — is *valid in the first place.*

Of the examples (RC-Ex1), (RC-Ex2), (RC-Ex3) cited above, (RC-Ex3) is especially closely related to the point of view taken in [CnstIII], §1. Indeed, in [CnstIII], §1, refers to the assertions of the RCS as the issue of showing the **"plurality of arithmetic holomorphic structures"** in inter-universal Teichmüller theory. Relative to this terminology, the central assertions of [CnstIII] may be summarized as follows:

(Js1) The *"plurality of arithmetic holomorphic structures"* in inter-universal Teichmüller theory is *not proven* in [IUTchI-IV].

(Js2) As a consequence of (Js1), the proofs of the main results of [IUTchI-IV], such as [IUTchIII], Corollary 3.12, are *incorrect.*

(Js3) A *correct proof* of [IUTchIII], Corollary 3.12, is given in [CnstIII] by applying the $p$-adic theory of tilts/untilts/Fontaine-Fargues curves.

In a word, *all three of these main assertions (Js1), (Js2), (Js3) are* **mathematically false**, as we explain in substantial detail in the discussion below.

First of all, with regard to (Js1), there is a confusion in the use of the phrase *"plurality of arithmetic holomorphic structures"*, especially, in the context of the analogy with Teichmüller deformations of Riemann surfaces, relative to the *two related [but by no means equivalent!] issues* of

(Js1-1) whether or not the *domain* and *codomain* Riemann surfaces in a *Teichmüller deformation* are required to be **non-isomorphic**;

(Js1-2) whether or not, when considering holomorphic isomorphisms between the *domain* and *codomain* Riemann surfaces as in (Js1-1), one imposes a certain **compatibility** condition, namely, that one requires such a holomorphic isomorphism to *induce the same map on underlying topological surfaces* as the Teichmüller deformation under consideration.

That is to say, even in the case of the *most classical and fundamental type of Teichmüller deformation* discussed in (RC-Ex3), the *non-isomorphicity* condition of (Js1-1) is *not satisfied*. Nevertheless, because in this context, one typically only considers holomorphic isomorphisms that are subject to the *compatibility* condition of (Js1-2), one conventionally regards, in classical complex Teichmüller theory, the various holomorphic structures arising from Teichmüller deformations as in (RC-Ex3) as **distinct**, hence, in particular, as constituting a **"plurality"** of holomorphic structures.

As explained in detail in [EssLgc], Example 3.5.3, (vi) [cf., especially, the discussion of (RCRS1), (RCRS2), (RCRS3), (RC$\Theta$1), (RC$\Theta$2), (RC$\Theta$3)], there is a *very precise correspondence* between this situation for Riemann surfaces and the situation surrounding the *domain and codomain [*$\Theta^{\pm \mathrm{ell}}$*NF-]Hodge theaters* of the **$\Theta$-link** in inter-universal Teichmüller theory. That is to say, relative to the analogous *compatibility* condition [cf. (Js1-2)], the **"plurality"** of arithmetic holomorphic structures is *valid* in inter-universal Teichmüller theory and indeed is an *immediate consequence of the definition of a ring* [i.e., in essence, the property observed in effect in (RC-Ex2) that the multiplicative map given by raising to the $N$-th power, for $N$ an integer $\geq 2$, on an integral domain of characteristic zero, is *not a ring homomorphism*] — cf. the discussion surrounding [EssLgc], Example 3.5.3, (vi), (RCRS1), (RCRS2), (RC$\Theta$1), (RC$\Theta$2).

Moreover, by considering *tilts*, one can indeed construct a situation [cf. [EssLgc], Example 3.5.3, (vi), (RCRS3), (RC$\Theta$3)] that is *roughly reminiscent of the situation* surrounding the domain and codomain of the $\Theta$-link in inter-universal Teichmüller theory [cf. [EssLgc], Example 3.5.3, (iv), (TltSim)], and which, moreover, satisfies the *non-isomorphicity* condition of (Js1-1), but which is, however, **completely useless** from the point of constructing a theory that is structurally similar to inter-universal Teichmüller theory, on account of the **numerous** and **quite fundamental structural differences** between this tilt-based construction and the corresponding constructions in inter-universal Teichmüller theory [cf. [EssLgc], Example 3.5.3, (iv), (vii), (TltDf1), (TltDf2), (TltDf3), (TltDf4), (TltDf5), (TltDf6), (TltDf7)].

With regard to (Js2), we observe [cf. [EssLgc], Example 3.5.3, (vi), (RC$\Theta$lg)] that the *"plurality"* of arithmetic holomorphic structures discussed above [i.e., relative to a *compatibility* condition analogous to the condition of (Js1-2)], which is *valid* in inter-universal Teichmüller theory and indeed is an *immediate consequence of the definition of a ring*, is in fact *never logically applied* in the development or proofs of the main results [such as, for instance, [IUTchIII], Theorem 3.11, or [IUTchIII], Corollary 3.12] of inter-universal Teichmüller theory. That is to say, even if one takes the position that one does not know whether or not the *"plurality"* of arithmetic holomorphic structures in *this* sense holds, there is *no effect whatsoever on the essential logical structure of the development or proofs of the main results* [such as, for instance, [IUTchIII], Theorem 3.11, or [IUTchIII], Corollary 3.12] of inter-universal Teichmüller theory. Put another way, the only effect of taking such a position is that it implies that there is a possibility that the theory involves a sort of **"overkill"**, i.e., that one is possibly doing more than is in fact necessary in order to prove the desired results.

Next, we consider (Js3). Tracing back through the *proofs of the main results* of [CnstIII], [CnstIV], starting with the *final conclusion* concerning the *ABC/Szpiro inequality* in [CnstIV], Theorem 7.2.1, one finds the following *chain of main implications*:

$$\begin{aligned} \text{[CnstIV], Theorem 7.2.1} \quad &\Longleftarrow \quad \text{[CnstIV], Theorem 7.1.1} \\ &\Longleftarrow \quad \text{[CnstIV], Theorem 6.1.1} \\ &\Longleftarrow \quad \text{[CnstIV], Theorem 6.10.1} \\ &\Longleftarrow \quad \text{[CnstIII], Corollary 9.11.1.1} \\ &\Longleftarrow \quad \text{[CnstIII], Theorem 9.11.1} \end{aligned}$$

[where we note that it is not clear whether or not the number "9.11..." assigned by the author to these key results in [CnstIII] was *purely coincidental* or a consequence of some sort of *sense of rhetoric or humor* that lies beyond my understanding].

In a word, the *central and quite fundamental problem* with the proof of [CnstIII], Theorem 9.11.1, which in fact underlies *all of the main results of [CnstIII], [CnstIV]* lies in the fact that

(LcGlIq) the argument given to prove [CnstIII], Theorem 9.11.1 [which is in fact entirely similar to the argument used in the final portion of the proof of [CnstIII], Theorem 7.3.1, or the proof of [CnstIII], Theorem 9.9.1] reduces the proof of a **global inequality** to the verification of **local inequalities** [i.e., at each prime of a number field], which are then *summed over* in order to obtain the global inequality.

Here, we recall that it is well-known — and indeed can be easily verified by considering *abc triples* of the form $(1, p^n, 1+p^n)$, for $p$ a prime number and $n$ an arbitrarily large positive integer — that

(EssGlIq) **any essentially global inequality** — i.e., such as the *ABC/Szpiro inequalities* or [IUTchIII], Corollary 3.12 — can **never** be obtained in this way, i.e., as a result of *summing up local inequalities at each prime of a number field.*

Indeed, it follows immediately from the computations in the proofs of [IUTchIV], Theorem 1.10 [which are summarized in Step (viii) of this proof]; [IUTchIV], Corollary 2.2, (ii), as well as the corresponding portions of the proofs of [ExpEst], Theorem 5.1; [ExpEst], Corollary 5.2, that if

· one collects the various terms in the log-volumes that appear in the *global inequality* of [IUTchIII], Corollary 3.12, and then multiplies by a suitable positive real number, and, moreover,

· one restricts one's attention, for simplicity, to abc triples/elliptic curves defined over a number field which is a *quadratic imaginary field*,

then the *local portion* of this global log-volume inequality at a *nonarchimedean valuation* $v$ of residue characteristic $p_v$ of the number field under consideration at which the elliptic curve under consideration has bad multiplicative reduction is of the form

$$\alpha \cdot \log(q_v) \ \leq \ \beta$$

— where we write $q_v$ for the $q$-parameter of the elliptic curve under consideration at $v$ and $\log(q_v)$ for the unique positive rational number $\lambda$ such that $p_v^{\lambda}/q_v$ is a $p_v$-adic unit; $\alpha \in A$ for some compact subset $A$ of the set of positive real numbers that is *independent* of the moduli of the elliptic curve under consideration; $\beta \in B$ for some compact subset $B$ of the set of real numbers that is *independent* of the moduli of the elliptic curve under consideration. In particular, if one considers *abc triples* of the form $(1, p^n, 1 + p^n)$ at a nonarchimedean valuation $v$ such that $p_v = p$, then $\alpha \cdot \log(q_v)$ is bounded below by a positive real multiple $\gamma \cdot n$ of $n$ [where the positive real number $\gamma$ is *independent* of $n$], hence $\to +\infty$ as $n \to +\infty$, thus *contradicting* the inequality of the above display.

More generally, at least as far as I could see, the *only nontrivial "global" objects* that appear in the theory developed in [CnstIII], [CnstIV] consist of *collections of local objects*, i.e., one object at each prime of a number field. Put another way,

(EssGl) **no essentially global phenomena** appear anywhere in the theory of [CnstIII], [CnstIV].

In this context, we recall that, by contrast, the *essentially global nature* of the *arithmetic line bundles* treated in [IUTchI-IV] may be seen in the subtle correspondence between **additive** and **multiplicative** representations of arithmetic line bundles, which involves, in an essential way, the **global ring structure** of a number field and, in particular, the way in which the relations arising from this global ring structure necessarily involve **all** of the **primes** of the number field [cf. [AbsTopIII], Remark 5.10.2, (iv); [IUTchII], Remark 4.11.2, (iii); [IUTchIII], Remark 3.6.2, (i)]. This correspondence between additive and multiplicative structures is closely related, in inter-universal Teichmüller theory, to the *delicate interplay* between **Frobenius-like** and **étale-like** structures, which is in turn closely related to the various **coricity**, **symmetry**, and **commutativity** properties surrounding the log-theta-lattice, all of which play a *fundamental role* in the *essential logical structure* of inter-universal Teichmüller theory [cf. the discussion of [EssLgc], §3.1, §3.2, §3.3, §3.4; [EssLgc], Example 3.2.2].

Here, we note that although the terms *"Frobenius-like"* and *"étale-like"* appear in [CnstIII], §8.4, the discussion involving these terms in [CnstIII], §8.4, *never* *mentions* these fundamental *coricity*, *symmetry*, and *commutativity* properties and, in particular, bespeaks the *apparently quite profound ignorance*, on the part of Joshi, of the *fundamental role* that such properties play in the *essential logical structure* of inter-universal Teichmüller theory, especially in the proof of [IUTchIII], Corollary 3.12 [cf. the discussion of [EssLgc], §3.10, §3.11]. Indeed, these properties are entirely irrelevant to the argument given in the proof of [CnstIII], Theorem 9.11.1.

Finally, in passing, we note *yet another fundamental problem* with the statement [i.e., the *chain of inequalities* in the first display of the statement] and proof of [CnstIII], Theorem 9.11.1:

(HllVl) the inequality concerning the volume of the region obtained *after passing to the hull* is obtained as a consequence of a **stronger inequality** concerning the volume of the region that arises *before passing to the hull*.

Here, the issue of finding a *direct contradiction* to such a *stronger inequality* is somewhat subtle and, at the time of writing, at least to my knowledge, strictly speaking, *inconclusive*. On the other hand, from the point of view of the *essential logical structure* of the argument in [IUTchIII] that is actually used to prove the inequality of [IUTchIII], Corollary 3.12, we recall that this inequality *cannot be concluded* until one completes the **closed loop** that is obtained by, in particular, *passing to the hull* [cf. the discussion of [EssLgc], §3.10, (Stp4), (Stp5), (Stp6), (Stp7), (Stp8)]. Moreover, if one is willing to ignore *certain technicalities* at the *archimedean primes* and the *nonarchimedean primes* that divide the rational prime 2, then a *stronger inequality* as in (HllVl) **contradicts** the existence of certain well-known types of *sequences of abc triples* due to *Masser* and others [cf. the discussion of [IUTchIV], Remark 1.10.5, (ii)].

That is to say, a *stronger inequality* as in (HllVl) would imply that, in Step (v) of the proof of [IUTchIV], Theorem 1.10, one could apply the *first displayed inequality*, of [IUTchIV], Proposition 1.4, (iii) — i.e., rather than the *second displayed inequality*, of [IUTchIV], Proposition 1.4, (iii), as is actually done — which would mean that one could *replace* the terms that are *linear in* $l$ in the inequalities of the final display in the statement of [IUTchIV], Theorem 1.10, by terms that are *linear in* $\log(l)$. Combining this strengthening of the inequalities of [IUTchIV], Theorem 1.10, with a suitably modified version of the estimates of [IUTchIV], Corollary 2.2, (ii), would yield a *contradiction* to certain well-known types of *sequences of abc triples* due to *Masser* and others, that is to say, if one is willing to ignore *certain technicalities* at the *archimedean primes* and the *nonarchimedean primes* that divide the rational prime 2. Here, we note in passing that these technicalities at the *nonarchimedean primes* that divide the rational prime 2 were in fact resolved in [ExpEst], and that it is *strongly expected* that the remaining technicalities at the *archimedean primes* will be resolved in an enhanced version of inter-universal Teichmüller theory that is currently under development.

Thus, in summary, although, strictly speaking, the existence of such *technicalities* means that, at a completely rigorous level, the *stronger inequality* of (HllVl) does *not* in fact yield an immediate contradiction, it is nevertheless *strongly expected*, especially in light of work in progress on various enhanced versions of inter-universal Teichmüller theory, that such technicalities are *inessential* and can be *overcome*, i.e., which would imply that the *stronger inequality* of (HllVl) does indeed imply a *numerical contradiction* in an entirely rigorous sense.

## Section 3: Further discussion

The discussion of §2 may be understood as an exposition of my understanding, at the time of writing, of the *mathematical content* underlying the *general overview* given in §1. On the other hand, since my understanding of the *p-adic theory of tilts/untilts/Fontaine-Fargues curves* [which lies beyond my range of specialty] is quite limited and superficial, I would certainly be open to further discussions concerning this theory, in the context of the mathematical content exposed in §2, with experts in the field. More generally, I would like to take this opportunity to state explicitly that I would be *open to* — and indeed, in many [though not necessarily all!] cases, *welcome* — the possibility of conducting **further discussions** concerning the content of the present report with mathematicians who belong to at least one of the following *categories*:

(MthCat1) mathematicians who are experts in **inter-universal Teichmüller theory** or **related anabelian geometry**, in the sense that they have *published paper(s) proving nontrivial result(s)* in these fields in *internationally recognized mathematical journal(s)*;

(MthCat2) mathematicians who are experts in the ***p*-adic theory** surrounding **tilts/untilts/Fontaine-Fargues curves**, again in the sense that they have *published paper(s) proving nontrivial result(s)* in these fields in *internationally recognized mathematical journal(s)*;

(MthCat3) senior mathematicians in **arithmetic geometry**, with the rank of **full professor** at a **world-class center of research**, who may not belong to (MthCat1) or (MthCat2), but who are nonetheless widely recognized as **leading researchers** in their field of specialty.

Here, it should be noted that Joshi does not belong to any of these categories. Indeed, the fact that he does not belong to (MthCat1) or (MthCat2) is closely related to the numerous issues discussed in the present report. Moreover, since the *technical core* of Joshi's series of preprints revolves around the *p-adic theory* of *tilts/untilts/Fontaine-Fargues curves* [cf. (MthCat2), i.e., a class of mathematicians to which neither Joshi nor I belong!], it seems essentially self-evident that the *next natural step*, with regard to the possibility of pursuing *further discussions* concerning the mathematical content of Joshi's series of preprints, must involve some sort of *interaction between (MthCat2) mathematicians and myself [or perhaps other (MthCat1) mathematicians]*. As a result, over the past few months, I have made *quite substantial efforts* to contact (MthCat2) mathematicians who were aware of Joshi's series of preprints to obtain information from them concerning their understanding of the mathematical content of this series of preprints. The only responses, however, that I was able to extract from such (MthCat2) mathematicians were to the effect that

(MC2Rsp) it was *obvious* from their point of view, i.e., from the point of view of the theory surrounding *tilts/untilts/Fontaine-Fargues curves*, that Joshi's series of preprints did **not contain any meaningful mathematical content**, and that, since they were busy with their own research projects, they were simply not interested in spending any more time or effort examining/discussing Joshi's series of preprints.

Interestingly enough, the *mathematical content* that was pointed out by (MthCat2) mathematicians as justification for the evaluation of (MC2Rsp) was *essentially identical* to the mathematical content discussed above in (EssGlIq), (EssGl) [i.e., which arose *independently* from my own investigations, as well as, again *independently*, from the investigations of other (MthCat1) mathematicians], namely, the observation that

(MC2EssGl) the theory developed in Joshi's series of preprints essentially only concerns *"global data"* that consists *solely of collections of local data*, where the "local data" was *not particularly novel*, but rather based on essentially known mathematical constructions, i.e., in short that there was **no meaningful essentially global content** to the theory.

Finally, it should be noted that previous [and indeed quite substantial] attempts, both on my part and on the part of other (MthCat1) mathematicians, to engage in discussions with Joshi for the purpose of explaining aspects of inter-universal Teichmüller theory concerning which he appeared to be confused *failed to result in a productive dialogue*.

## Bibliography

[EMSCOP] European Mathematical Society, *Code of Practice*, approved by the EMS Executive Committee (October 2012), available at the following URL:
`https://euromathsoc.org/code-of-practice`

[AbsTopIII] S. Mochizuki, Topics in Absolute Anabelian Geometry III: Global Reconstruction Algorithms, *J. Math. Sci. Univ. Tokyo* **22** (2015), pp. 939-1156.

[IUTchI] S. Mochizuki, Inter-universal Teichmüller Theory I: Construction of Hodge Theaters, *Publ. Res. Inst. Math. Sci.* **57** (2021), pp. 3-207.

[IUTchII] S. Mochizuki, Inter-universal Teichmüller Theory II: Hodge-Arakelov-theoretic Evaluation, *Publ. Res. Inst. Math. Sci.* **57** (2021), pp. 209-401.

[IUTchIII] S. Mochizuki, Inter-universal Teichmüller Theory III: Canonical Splittings of the Log-theta-lattice, *Publ. Res. Inst. Math. Sci.* **57** (2021), pp. 403-626.

[IUTchIV] S. Mochizuki, Inter-universal Teichmüller Theory IV: Log-volume Computations and Set-theoretic Foundations, *Publ. Res. Inst. Math. Sci.* **57** (2021), pp. 627-723.

[ExpEst] S. Mochizuki, I. Fesenko, Y. Hoshi, A. Minamide, W. Porowski, Explicit Estimates in Inter-universal Teichmüller Theory, *Kodai Math. J.* **45** (2022), pp. 175-236.

[Rpt2023] S. Mochizuki, *Brief Report on the Current Situation Surrounding Inter-universal Teichml̈ler Theory (IUT)*, available at the following URL:
`http://www.kurims.kyoto-u.ac.jp/~motizuki/2023-08%20Brief%20report%20on%20the%20current%20situation%20surrounding%20inter-universal%20Teichmuller%20theory%20(IUT).pdf`

[Alien] S. Mochizuki, The Mathematics of Mutually Alien Copies: from Gaussian Integrals to Inter-universal Teichmüller Theory, *Inter-universal Teichmuller*

*Theory Summit 2016*, *RIMS Kōkyūroku Bessatsu* **B84**, Res. Inst. Math. Sci. (RIMS), Kyoto (2021), pp. 23-192.

[EssLgc] S. Mochizuki, On the essential logical structure of inter-universal Teichmüller theory in terms of logical AND "$\wedge$"/logical OR "$\vee$" relations: Report on the occasion of the publication of the four main papers on inter-universal Teichmüller theory, RIMS Preprint 1968 (November 2022); updated version available at the following URL:
`http://www.kurims.kyoto-u.ac.jp/~motizuki/Essential%20Logical%20Structure%20of%20Inter-universal%20Teichmuller%20Theory.pdf`